basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2024

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 25 pages./
Hierdie nasienriglyne bestaan uit 25 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

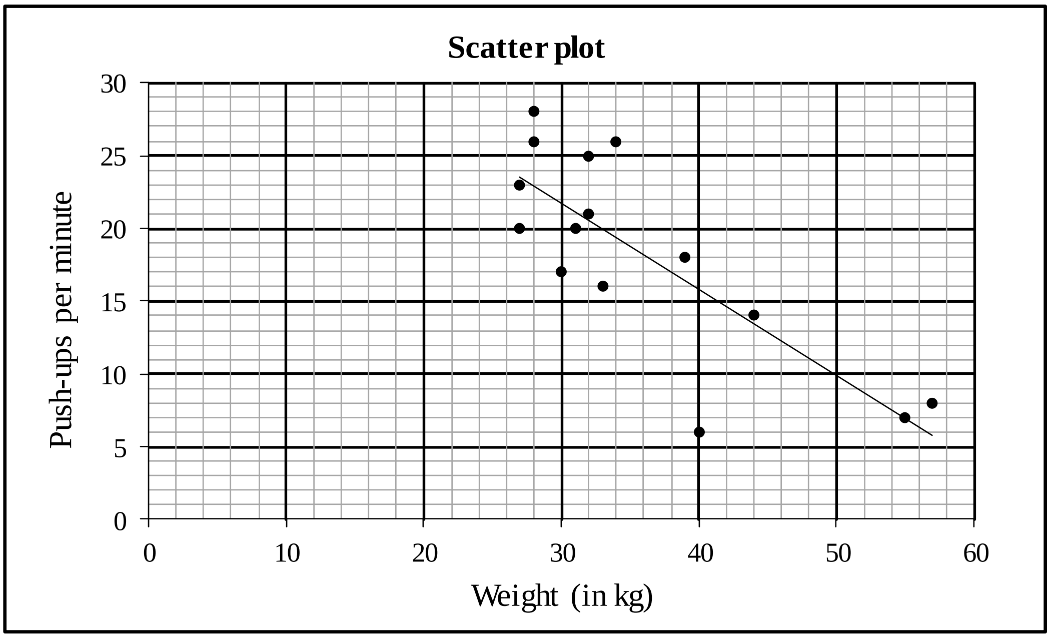
LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • MEETKUNDE	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

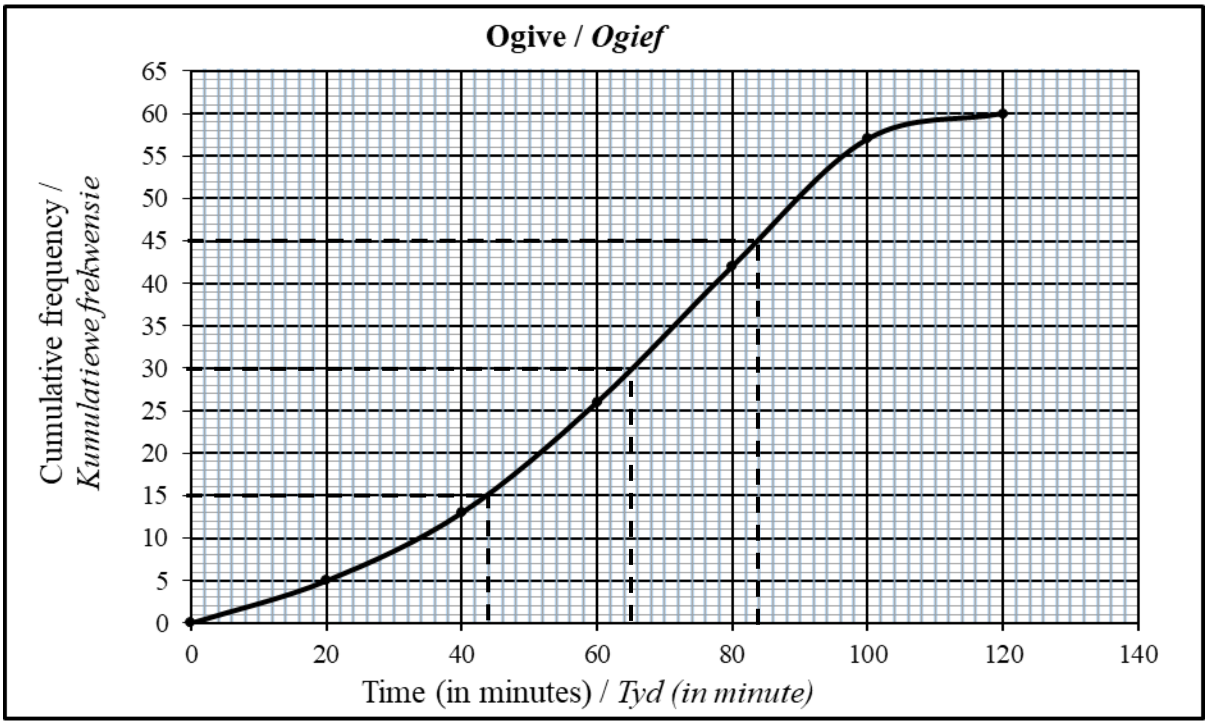
QUESTION/*VRAAG* 1

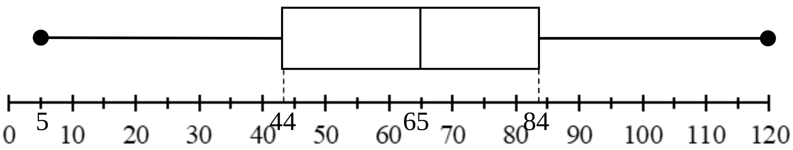
Weight (in kg) (x)	34	32	40	27	33	28	27	55	39	44	30	57	28	32	31
Number of push-ups per minute (y)	26	21	6	20	16	26	23	7	18	14	17	8	28	25	20



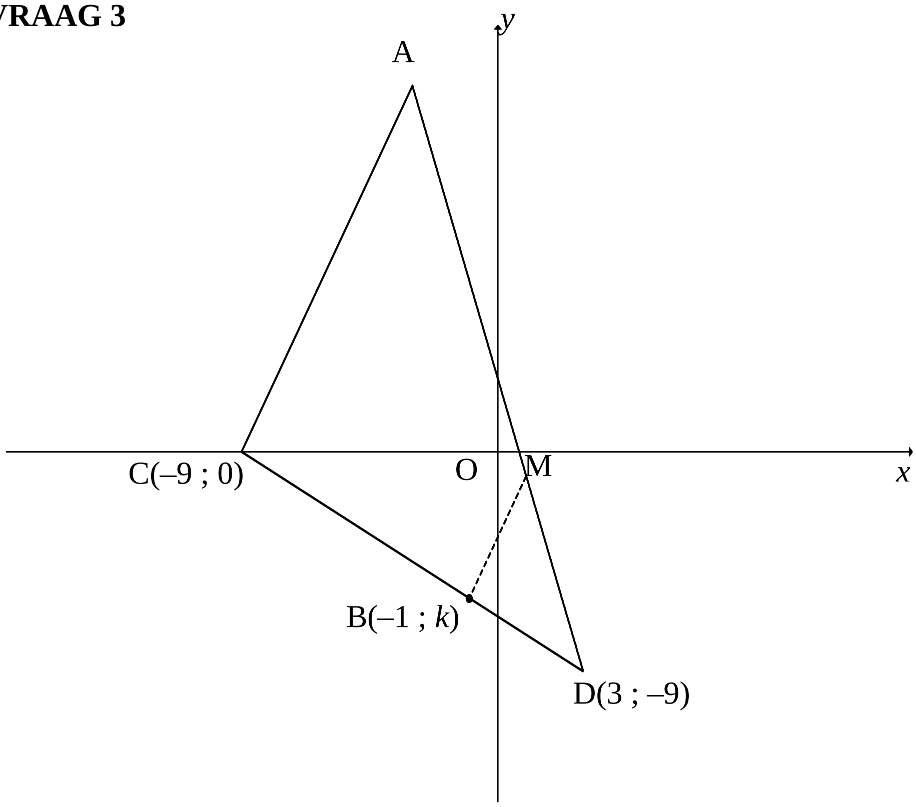
1.1	$a = 39{,}456001\ldots$ $b = -0{,}590018\ldots$ $\hat{y} = 39{,}46 - 0{,}59x$ CORRECT ANSWER ONLY: FULL MARKS	✓ $a = 39{,}46$ ✓ $b = -0{,}59$ ✓ equation (3)
1.2	$r = -0{,}8$	✓(A) $-0{,}8$ (1)
1.3	$y = 39{,}46 - 0{,}59(29)$ $y = 22{,}35$ **OR/*OF*** $y = 22{,}35$ (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$\bar{y} = 18{,}33$	✓(A) 18,33 (1)
1.5	The increase in the number of push-ups will have **no influence**. The standard deviation **stays the same**.	✓ no influence **OR** standard deviation remains the same *geen verandering / bly dieselfde* (1)
1.6	6 is furthest y-value below the least squares regression line. An increase of 10 push-ups will get the team member to (40 ; 16), the minimum number of push-ups for a player weighing 40kg.	✓ 6 ✓ difference is 10 (2)
		[10]

QUESTION/*VRAAG* 2



2.1	Median = 65	✓ 65 (1)
2.2	$Q_1 = 44$	✓ 44 (1)
2.3	IQR = 84 – 44 = 40	✓ 84 ✓ IQR (2)
2.4		✓ box ✓(A) whiskers ending at 5 & 120 (2)
2.5	Number of employees who qualify = 34 % of employees who qualify $= \frac{34}{60} \times 100$ = 56,67% of the employees **OR/*OF*** Number of employees who qualify = 35 % of employees who qualify $= \frac{35}{60} \times 100$ = 58,33% of the employees	✓ 34 ✓ answer (2) ✓ 35 ✓ answer (2)
2.6	Number of intervals = 3 Time allowed to work from home = 3(30 minutes) = 90 minutes **OR/*OF*** 1,5 hours	✓ 3 ✓ answer (2)
		[10]

QUESTION / VRAAG 3

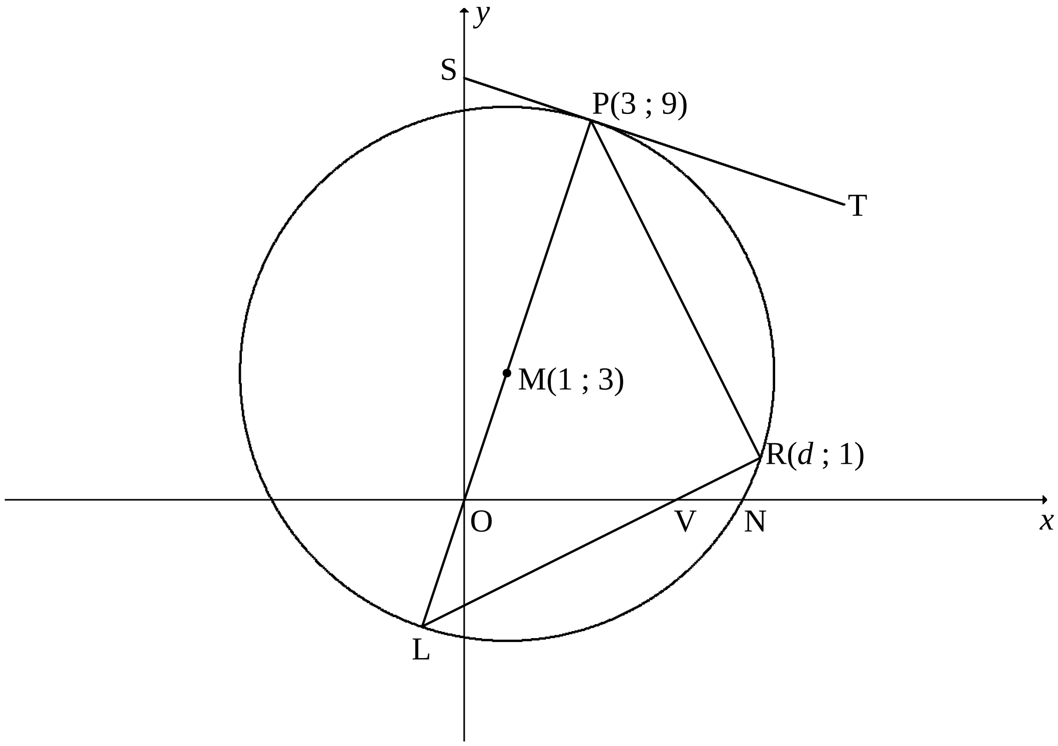


3.1	$m_{DC}=\frac{-9-0}{3-(-9)}$ **OR/*OF*** $m_{DC}=\frac{0-(-9)}{-9-3}$ $m_{DC}=-\frac{3}{4}$ $m_{DC}=-\frac{3}{4}$	✓ correct substitution of $D(3;-9)$ & $C(-9;0)$ into gradient formula ✓ answer (2)
3.2	Equation of DC: $0=-\frac{3}{4}(-9)+c$ **OR/*OF*** $y-0=-\frac{3}{4}(x-(-9))$ $c=\frac{-27}{4}$ or $-6\frac{3}{4}$ $y=-\frac{3}{4}(x+9)$ $y=-\frac{3}{4}x-\frac{27}{4}$ $y=-\frac{3}{4}x-\frac{27}{4}$	✓ correct substitution of $C(-9;0)$ or $D(3;-9)$ into equation of line ✓ answer (2)
3.3	$k=-\frac{3}{4}(-1)-\frac{27}{4}$ **OR/*OF*** $\frac{k-(-9)}{-1-3}=\frac{-3}{4}$ **OR/*OF*** $\frac{k-0}{-1-(-9)}=\frac{-3}{4}$ $k=\frac{3}{4}-\frac{27}{4}$ **OR/*OF*** $k+9=3$ **OR/*OF*** $k=-\frac{3}{4}(8)$ $k=-6$ $k=-6$ $k=-6$	✓ substitution of $B(-1;k)$ (1)
3.4	$DC=\sqrt{(3+9)^2+(-9-0)^2}$ DC = 15 units	✓ correct substitution of $D(3;-9)$ & $C(-9;0)$ into distance formula ✓ answer (2)

3.5	$DB=\sqrt{(3-(-1))^2+(-9-(-6))^2}$ $DB = 5$ $\therefore \frac{DB}{DC}=\frac{5}{15}=\frac{1}{3}$	✓ DB = 5 ✓ answer (2)
3.6	$\frac{DM}{DA}=\frac{DB}{DC}=\frac{1}{3}$ $\frac{\text{Area }\Delta MBD}{\text{Area }\Delta ACD}=\frac{\frac{1}{2}(DM)(DB)\left(\sin\hat{D}\right)}{\frac{1}{2}(DA)(DC)\left(\sin\hat{D}\right)}$ $=\frac{1}{3}\times\frac{1}{3}$ $=\frac{1}{9}$	✓ $\frac{DM}{DA}=\frac{DB}{DC}$ ✓ correct use of area rule ✓ subst. for $\frac{BD}{DC}$ and $\frac{DM}{DA}$ into correct formula ✓ answer (4)
3.7	$y = -4x + c$ $m_{AD}=-4$ $-9=-4(3)+c$ **OR/*OF*** $\frac{y+9}{x-3}=-4$ $c = 3$ $y+9=-4x+12$ $y=-4x+3$ $y=-4x+3$ $(x-3)^2+(y+9)^2=612$ $(x-3)^2+(-4x+3+9)^2=\left(\sqrt{612}\right)^2$ $(x-3)^2+(-4x+12)^2=612$ $x^2-6x+9+16x^2-96x+144=612$ $17x^2-102x-459=0$ $x^2-6x-27=0$ $(x-9)(x+3)=0$ $x=9$ or $x=-3$ N/A $y = -4(-3) + 3$ $y = 15$ $A(-3;15)$	✓ correct substitution of $m_{AD}=-4$ and D(3 ; –9) ✓ $(x-3)^2+(y+9)^2=612$ ✓ substitution of equation AD into distance formula ✓ standard form ✓ x values with rejection ✓ y coordinate (6)

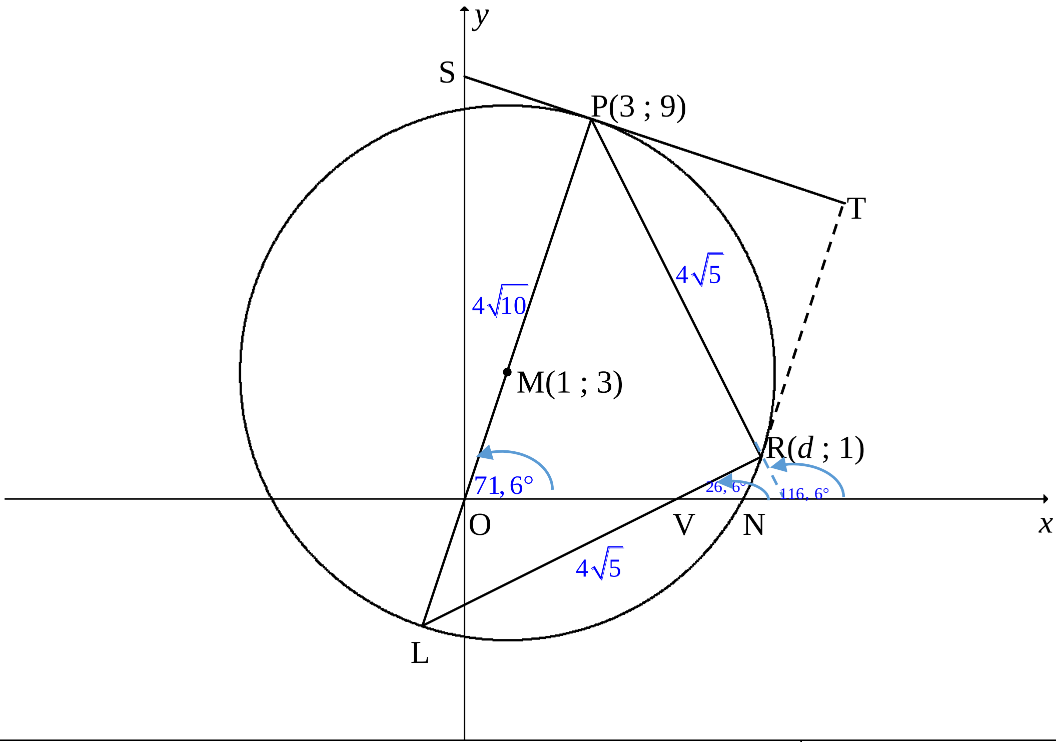
	OR/*OF* $-9=-4(3)+c$ $c=3$ $y=-4x+3$ N(0 ; 3) ND $=\sqrt{(3-0)^2+(-9-3)^2}$ $=3\sqrt{17}$ AD $=6\sqrt{17}$ $\text{ND}=\frac{1}{2}\text{AD}$ N is the midpoint of AD A(–3 ; 15)	**OR/*OF*** ✓ correct substitution of $m_{\text{AD}}=-4$ and D(3 ; –9) ✓ N(0 ; 3) ✓ substitution into distance formula to calculate ND ✓ $\text{ND}=\frac{1}{2}\text{AD}$ ✓ x – value ✓ y – value (6)
		[19]

QUESTION/*VRAAG* 4



4.1	L(–1 ; –3)	✓ $x = -1$ ✓ $y = -3$ (2)
4.2	$m_{MP} = \frac{9-3}{3-1}$ $m_{MP} = 3$ $m_{ST} = -\frac{1}{3}$ $9 = -\frac{1}{3}(3) + c$ $c = 10$ $y = -\frac{1}{3}x + 10$ **OR/*OF*** $y - 9 = -\frac{1}{3}(x-3)$ $y - 9 = -\frac{1}{3}x + 1$ $y = -\frac{1}{3}x + 10$	✓ $m_{MP} = 3$ ✓ $m_{ST} = -\frac{1}{m_{MP}}$ ✓ substitution of m_{ST} & P(3; 9) into equation of a line ✓ equation of tangent ST (4)
4.3	$(x-1)^2 + (y-3)^2 = r^2$ $(3-1)^2 + (9-3)^2 = r^2$ $r^2 = 40$ $(x-1)^2 + (y-3)^2 = 40$ $x^2 - 2x + 1 + y^2 - 6y + 9 = 40$ $x^2 + y^2 - 2x - 6y - 30 = 0$	✓ $(3-1)^2 + (9-3)^2 = r^2$ ✓ value of r^2 ✓ LHS of equation of circle ✓ expanding LHS (4)

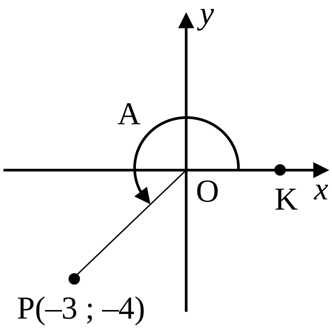
4.4	$d^2+(1)^2-2d-6(1)-30=0$ $d^2-2d-35=0$ $(d-7)(d+5)=0$ $d=7$ or $d=-5$ $\therefore d=7$	✓ $d^2+(1)^2-2d-6(1)-30=0$ ✓ standard form (2)
	OR/*OF*	**OR/*OF***
	$(x-1)^2+(y-3)^2=40$ $(d-1)^2+(1-3)^2=40$ $(d-1)^2=36$ $d-1=6$ or $d-1=-6$ $d=7$ or $d=-5$ $\therefore d=7$	✓ $(d-1)^2+(1-3)^2=40$ ✓ standard form (2)
	OR/*OF*	**OR/*OF***
	$P\hat{R}L=90^\circ$ (∠ in semi-circle) $\frac{9-1}{3-d}\times\frac{1-(-3)}{d-(-1)}=-1$ $d^2-2d-35=0$ $(d-7)(d+5)=0$ $d=7$ or $d=-5$ $\therefore d=7$	✓ $m_{PR}\times m_{RL}=-1$ ✓ standard form (2)

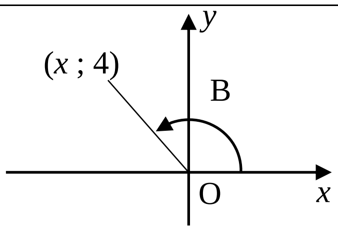


4.5	$m_{PO} = 3$	
	$\therefore \tan P\hat{O}V = 3$	✓ $\tan P\hat{O}V = m_{PO}$
	$P\hat{O}V = 71{,}565...°$	✓ $P\hat{O}V$
	$m_{RL} = \frac{1-(-3)}{7-(-1)}$	✓ m_{RL} using R(7 ; 1) & L
	$= \frac{1}{2}$	
	$\therefore \tan R\hat{V}N = \frac{1}{2}$	
	$R\hat{V}N = 26{,}565...°$	✓ $R\hat{V}N$
	$\hat{L} = 71{,}565...° - 26.565...°$ [ext. ∠ of Δ / *buite ∠ van Δ*]	
	$\hat{L} = 45°$	✓ answer
		(5)
	OR/*OF*	**OR/*OF***
	$\hat{R} = 90°$ [∠ in semi-circle / *∠ in 'n halwe sirkel*]	✓ $\hat{R} = 90°$
	$PR^2 = (3-7)^2 + (9-1)^2$	
	$PR = \sqrt{80} = 4\sqrt{5}$ units	✓ $PR = \sqrt{80} = 4\sqrt{5}$
	$PL^2 = (3-(-1))^2 + (9-(-3))^2$ **OR** $RL^2 = (7+1)^2 + (1+3)^2$	
	$PL = \sqrt{160} = 4\sqrt{10}$ $RL = \sqrt{80} = 4\sqrt{5}$	✓ length of PL **OR** RL
	$\sin \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\cos \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ **OR** $\tan \hat{L} = \frac{4\sqrt{5}}{4\sqrt{5}}$	✓ trig ratio of $\hat{L}$
	$\hat{L} = 45°$	✓ answer
		(5)

	OR/*OF* $PL=\sqrt{(3+1)^2+(9+3)^2}=\sqrt{160}=4\sqrt{10}$ $PR=\sqrt{(7-3)^2+(1-9)^2}=\sqrt{80}=4\sqrt{5}$ $LR=\sqrt{(7+1)^2+(1+3)^2}=\sqrt{80}=4\sqrt{5}$ $\cos L=\frac{80+160-80}{2\sqrt{80}\times\sqrt{160}}$ $\cos L=\frac{\sqrt{2}}{2}$ $\hat{L}=45°$	**OR/*OF*** ✓ length of PL ✓ $PR=\sqrt{80}=4\sqrt{5}$ ✓ length of LR ✓ substitution into the cos rule ✓ answer (5)
4.6	$m_{RM}=\frac{1-3}{7-1}$ $=-\frac{1}{3}$ $m_{RT}=3$ (tan ⊥ rad) $m_{PT}=-\frac{1}{3}$ $m_{RT}\times m_{PT}=-1$ PT ⊥ RT	✓ m_{RM} ✓ m_{RT} ✓ $m_{RT}\times m_{PT}=-1$ (3)
	OR/*OF* $m_{MR}=\frac{3-1}{1-7}$ $=-\frac{1}{3}$ $m_{PT}=-\frac{1}{3}$ [proved in Q4.2] $m_{PT}=m_{MR}$ ∴ PT ∥ MR $M\hat{R}T=90°$ [radius ⊥ tangent / *raaklyn* ⊥ *radius*] $P\hat{T}R=90°$ [co-int ∠s; PT ∥ MR/*ooreenkomst. ∠e*; PT ∥ MR] PT ⊥ RT	**OR/*OF*** ✓ PT ∥ MR ✓ $M\hat{R}T=90°$ ✓ $P\hat{T}R=90°$ (3)
	OR/*OF* $T\hat{P}R=\hat{L}=45°$ [tan-chord theorem/ ∠ *tussen raaklyn en koord*] $TP=TR$ [tans from common pt] $\therefore T\hat{P}R=T\hat{R}P=45°$ [∠s opp equal sides/ ∠*e teenoor gelyke sye*] $\therefore P\hat{T}R=90°$ [sum of ∠s in Δ / *binne ∠e van* Δ] PT ⊥ RT	**OR/*OF*** ✓ $T\hat{P}R=\hat{L}$ ✓ $T\hat{P}R=T\hat{R}P$ ✓ $P\hat{T}R=90°$ (3)
		[20]

QUESTION/*VRAAG* 5



5.1.1	$r = 5$ $\cos A = -\frac{3}{5}$	✓ $r = 5$ ✓ answer (2)
5.1.2	$\cos 2A = 2\cos^2 A - 1$ $= 2\left(-\frac{3}{5}\right)^2 - 1$ $= -\frac{7}{25}$ **OR/*OF*** $\cos 2A = \cos^2 A - \sin^2 A$ $= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$ **OR/*OF*** $\cos 2A = 1 - 2\sin^2 A$ $= 1 - 2\left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$	✓ substitution of $\cos A$ into double angle formula ✓ answer (2) ✓ substitution of $\cos A$ & $\sin A$ into double angle formula ✓ answer (2) ✓ substitution of $\sin A$ into double angle formula ✓ answer (2)
5.1.3	 $x = -3$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= \frac{12}{25} + \frac{12}{25}$ $= \frac{24}{25}$	✓ $x = -3$ ✓✓ substitution into the compound angle formula ✓ answer (4)

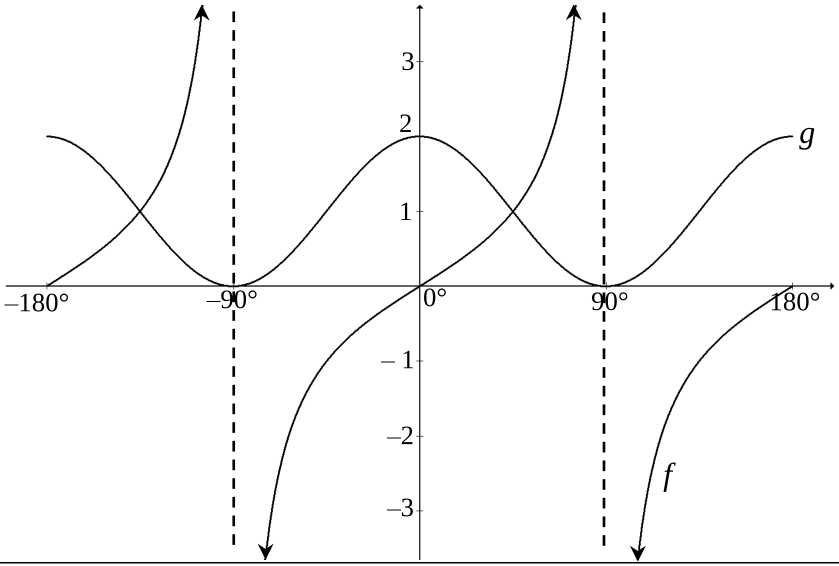
5.2	$\dfrac{\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2}$ $=\dfrac{2\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2.2}$ $=\dfrac{\sin(\alpha-90°)}{4}$ $=\dfrac{-\cos\alpha}{4}$ $=\dfrac{-p}{4}$ **OR/*OF*** $=-\dfrac{1}{4}p$	✓ multiply by $\dfrac{2}{2}$ ✓ double angle ✓ co function ✓ answer (4)
	OR/*OF*	**OR/*OF***
	$\dfrac{\cos\left(\frac{\alpha}{2}-45°\right)\sin\left(\frac{\alpha}{2}-45°\right)}{2}$ $=\dfrac{\left[\cos\frac{\alpha}{2}\cos 45°+\sin\frac{\alpha}{2}\sin 45°\right]\left[\sin\frac{\alpha}{2}\cos 45°-\cos\frac{\alpha}{2}\sin 45°\right]}{2}$ $=\dfrac{\left[\frac{\sqrt{2}}{2}\cos\frac{\alpha}{2}+\frac{\sqrt{2}}{2}\sin\frac{\alpha}{2}\right]\left[\frac{\sqrt{2}}{2}\sin\frac{\alpha}{2}-\frac{\sqrt{2}}{2}\cos\frac{\alpha}{2}\right]}{2}$ $=\dfrac{\frac{1}{2}\sin^2\frac{\alpha}{2}-\frac{1}{2}\cos^2\frac{\alpha}{2}}{2}$ $=\dfrac{-\frac{1}{2}\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)}{2}$ $=-\dfrac{\cos 2\left(\frac{\alpha}{2}\right)}{4}$ $=-\dfrac{\cos\alpha}{4}$ $=-\dfrac{1}{4}p$	✓ expansion ✓ special angles ✓ double angle ✓ answer (4)
		[12]

QUESTION/VRAAG 6

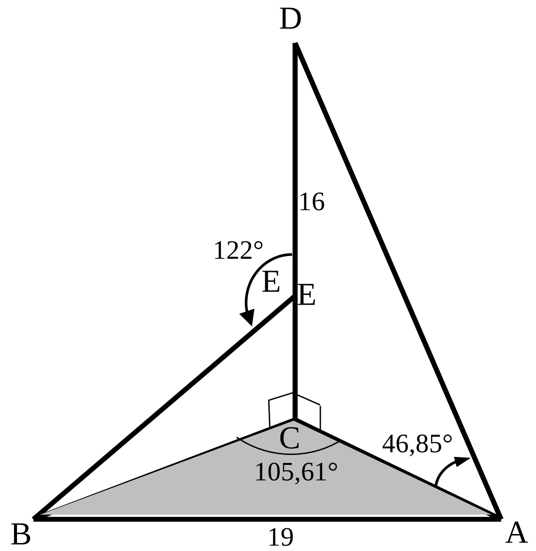
6.1.1	$\cos(x+y)=\cos(x-(-y))$ $=\cos x\cos(-y)+\sin x\sin(-y)$ $=\cos x\cos y-\sin x\sin y$	✓ $(x+y)=(x-(-y))$ ✓ correct expansion (2)
6.1.2	LHS $=\dfrac{\cos(90°-x)\cos y+\sin(-y)\cos(180°+x)}{\cos x\cos(360°+y)+\sin(360°-x)\sin y}$ $=\dfrac{(\sin x)\cos y+(-\sin y)(-\cos x)}{\cos x(\cos y)+(-\sin x)\sin y}$ $=\dfrac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$ $=\dfrac{\sin(x+y)}{\cos(x+y)}$ $=\tan(x+y)$ = RHS	✓ $\cos(90°-x)=\sin x$ ✓ $\sin(-y)=-\sin y$ ✓ $\cos(180°+x)=-\cos x$ ✓ $\cos(360°+y)=\cos y$ ✓ $\sin(360°-x)=-\sin x$ ✓ compound angle formulae (6)
6.2	$\sqrt{6\sin^2 x-11\cos(90°+x)+7}=2$ $6\sin^2 x-11\cos(90°+x)+7=4$ $6\sin^2 x-11(-\sin x)+7=4$ $6\sin^2 x+11\sin x+3=0$ $(3\sin x+1)(2\sin x+3)=0$ $\sin x=-\dfrac{1}{3}$ **OR/*OF*** $\sin x=-\dfrac{3}{2}$ ref$\angle=19{,}47°$ no solution $x=199{,}47°$ or $x=340{,}53°$	✓squaring both sides ✓ $\cos(90°+x)=-\sin x$ ✓ factors ✓ both equations ✓✓ answers (6)
6.3.1	$g(x)=\dfrac{4-8\sin^2 x}{3}$ $=\dfrac{4(1-2\sin^2 x)}{3}$ $=\dfrac{4\cos 2x}{3}$ Maximum value of cos 2*x* is 1 $\therefore$ maximum value of $g(x)=\dfrac{4}{3}$	✓ factors ✓ $\dfrac{4\cos 2x}{3}$ ✓ answer (3)

	OR/*OF* $4-8\sin^2 x$ is a maximum when $\sin^2 x$ is a minimum Minimum value of $\sin^2 x$ is 0 $\therefore$ max. value of $g(x)=\dfrac{4-8(0)}{3}$ $g(x)=\dfrac{4}{3}$ **OR/*OF*** $\sin x=\dfrac{-(0)}{2\left(-\frac{8}{3}\right)}$ $\sin x=0$ $\therefore$ max. value of $g(x)=\dfrac{4-8(0)}{3}$ $g(x)=\dfrac{4}{3}$	**OR/*OF*** ✓ min of $\sin^2 x=0$ ✓ $g(x)=\dfrac{4-8(0)}{3}$ ✓ answer (3) **OR/*OF*** ✓ $\sin x=\dfrac{-(0)}{2\left(-\frac{8}{3}\right)}$ ✓ $\sin x=0$ ✓ answer (3)
6.3.2	$x = 180°$	✓ 180° (1)
		[18]

QUESTION/*VRAAG* 7

7.1	$x = 90°$	✓ $x = 90°$ (1)
7.2	$x = -180°$ or $x \in (-90°\ ;\ 0°]$ **OR/*OF*** $x = -180°$ or $-90° < x \leq 0°$	✓✓ answer (2) ✓✓ answer (2)
7.3.1	180°	✓ answer (1)
7.3.2		✓ turning points on x-axis: $x = -90°\ ;\ 90°$ ✓ shape ✓ turning point on y-axis at (0 ; 2) (3)
7.4	$2\cos^3 x - \sin x = 0$ $2\cos^3 x = \sin x$ $2\cos^2 x = \frac{\sin x}{\cos x}$ $2\cos^2 x = \tan x$ $2\cos^2 x - 1 = \tan x - 1$ $\cos 2x + 1 = \tan x$ $x = 45° + k.180°;\ k \in Z$ **OR/*OF*** $2\cos^3 x - \sin x = 0$ $\cos x\left(2\cos^2 x - \tan x\right) = 0$ $\cos x = 0$ or $2\cos^2 x = \tan x$ not valid; $2\cos^2 x - 1 + 1 = \tan x$ $\cos 2x + 1 = \tan x$ $x = 45° + k.180°;\ k \in Z$	✓ $2\cos^2 x = \tan x$ ✓ $2\cos^2 x - 1 = \tan x - 1$ ✓ $\cos 2x + 1 = \tan x$ ✓ answer (4) **OR/*OF*** ✓ $2\cos^2 x = \tan x$ ✓ $2\cos^2 x - 1 + 1 = \tan x$ ✓ $\cos 2x + 1 = \tan x$ ✓ answer (4)
		[11]

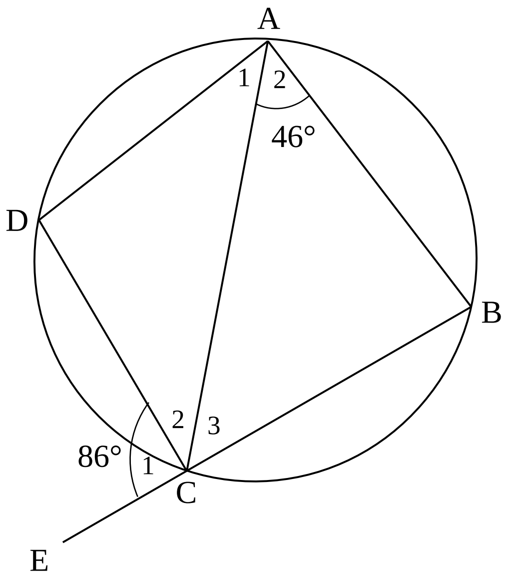
QUESTION/*VRAAG* 8



8.1	$\tan D\hat{A}C = \frac{DC}{AC}$ $AC = \frac{16}{\tan 46{,}85°}$ $AC = 15$ m	✓ correct subs into trig ratio ✓ answer (2)
8.2	$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC)\cos B\hat{C}A$ $(19)^2 = x^2 + (15)^2 - 2x(15)\cos 105{,}61°$ $x^2 + 8{,}07x - 136 = 0$ $x = \frac{-8{,}07 \pm \sqrt{(8{,}07)^2 - 4(1)(-136)}}{2(1)}$ $x = 8{,}30$ m or $x \neq -16{,}38$ m $B\hat{E}C = 58°$ $\tan B\hat{E}C = \frac{BC}{EC}$ $EC = \frac{8{,}3}{\tan 58°}$ $EC = 5{,}19$ m $DE = 10{,}81$ m **OR/*OF*** $E\hat{B}C = 32°$ $\tan E\hat{B}C = \frac{EC}{BC}$ $EC = 8{,}3\tan 32°$ $EC = 5{,}19$ m $DE = 10{,}81$ m	✓ correct subst. into cosine rule ✓ quadratic equation in std form ✓ correct subst. into quadratic formula ✓ length of BC ✓ size of $B\hat{E}C$ **OR/*OF*** $E\hat{B}C$ ✓ length of EC ✓ answer (7)

	OR/*OF* $\frac{\sin 105{,}61°}{19}=\frac{\sin C\hat{B}A}{15}$ $C\hat{B}A = 49{,}5°$ $B\hat{A}C = 24{,}89°$ $\frac{BC}{\sin 24{,}89°}=\frac{19}{\sin 105{,}61°}$ BC = 8,3 m $B\hat{E}C = 58°$ **OR/*OF*** $E\hat{B}C = 32°$ $\tan B\hat{E}C=\frac{BC}{EC}$; $\tan E\hat{B}C=\frac{EC}{BC}$ $EC=\frac{8{,}3}{\tan 58°}$; $EC = 8{,}3\tan 32°$ $EC = 5{,}19$ m ; $EC = 5{,}19$ m $DE = 10{,}81$ m ; $DE = 10{,}81$ m	**OR/*OF*** ✓ correct subst. into sine rule ✓ $B\hat{A}C$ ✓ correct subst. into sine formula ✓ length of BC ✓ size of $B\hat{E}C$ **OR/*OF*** $E\hat{B}C$ ✓ length of EC ✓ answer (7)
		[9]

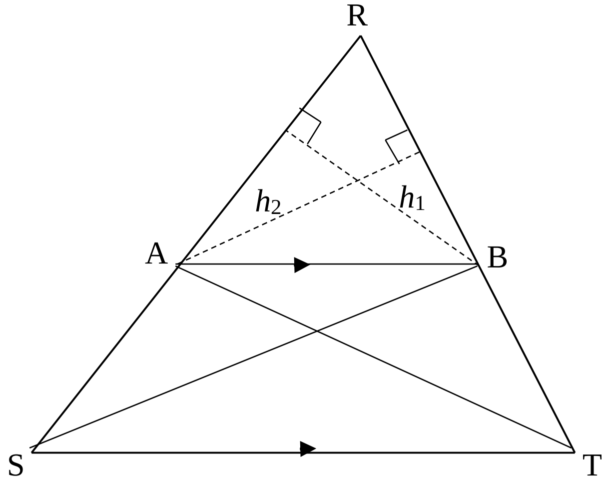
QUESTION/VRAAG 9



9.1	$\hat{A}_1 = 40°$ [ext. ∠ of a cyclic quad / *buite ∠ van kvh*]	✓ S ✓ R (2)
9.2	$\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\hat{D} = 100°$ [opp ∠s of cyclic quad / *teenoorst. ∠e van kvh*] $\therefore \hat{C}_2 = 40°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ ∴ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $A\hat{C}E = \hat{A}_2 + \hat{B}$ [ext ∠ of Δ / *buite ∠ van* Δ] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ ∴ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*] **OR/*OF*** $\hat{B} = 80°$ $\left[\hat{A}_1 = \frac{1}{2}\hat{B}\right]$ $\therefore \hat{C}_3 = 180° - 46° - 80°$ [sum of ∠s in Δ / *binne ∠e van* Δ] $\therefore \hat{C}_3 = 54°$ $\therefore \hat{C}_2 = 180° - 86° - 54°$ [∠s on a str. line / *∠e op 'n reguitlyn*] $\therefore \hat{C}_2 = 40°$ $\therefore \hat{C}_2 = \hat{A}_1 = 40°$ ∴ AD = DC [sides opp = ∠s / *sye teenoor gelyke ∠*]	✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S/R ✓ S ✓ R (4)
		[6]

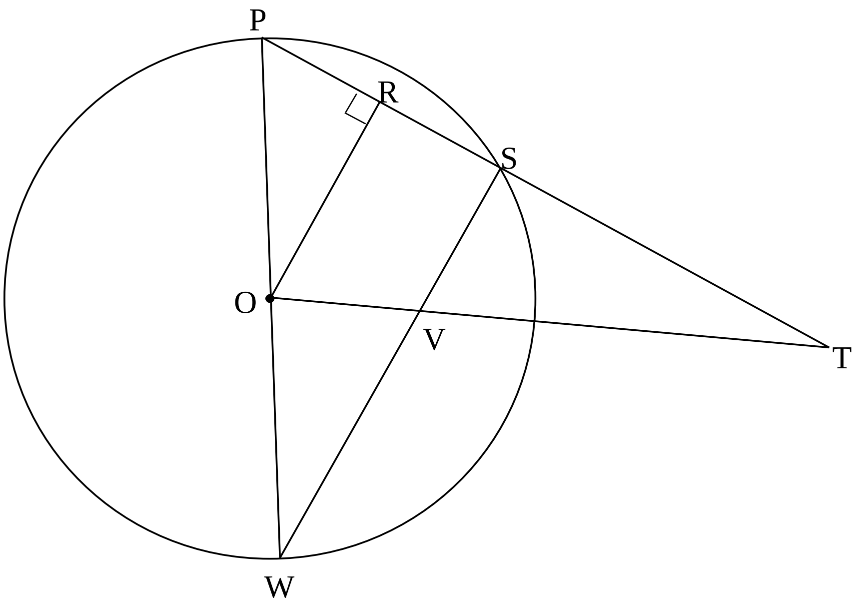
QUESTION/VRAAG 10

10.1



10.1	Construction: Join SB and TA and draw h_1 from B $\perp$ AR and h_2 from A $\perp$ RB *Konstruksie: Verbind SB en TA en trek h_1 vanaf B $\perp$ AR en h_2 vanaf A $\perp$ RB* Proof/*Bewys*: $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\frac{1}{2}\text{RA} \times h_1}{\frac{1}{2}\text{AS} \times h_1} = \frac{\text{RA}}{\text{AS}}$ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}} = \frac{\frac{1}{2}\text{RB} \times h_2}{\frac{1}{2}\text{BT} \times h_2} = \frac{\text{RB}}{\text{BT}}$ area ΔRAB = area ΔRAB [common/*gemeenskaplik*] But area ΔASB = area ΔABT [same base & height; AB ‖ ST/ *dies. basis & hoogte; AB ‖ST*] $\therefore \frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}}$ $\therefore \frac{\text{RA}}{\text{AS}} = \frac{\text{RB}}{\text{BT}}$	✓ construction ✓ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ASB}} = \frac{\frac{1}{2}\text{RA} \times h_1}{\frac{1}{2}\text{AS} \times h_1}$ ✓ $\frac{\text{RA}}{\text{AS}}$ ✓ $\frac{\text{area } \Delta\text{RAB}}{\text{area } \Delta\text{ABT}} = \frac{\text{RB}}{\text{BT}}$ ✓ S ✓ R (6)

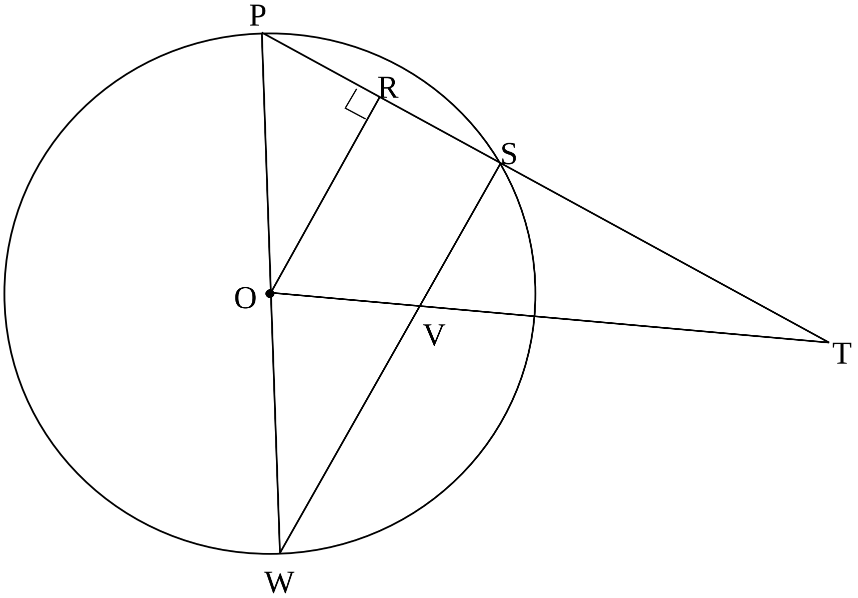
10.2



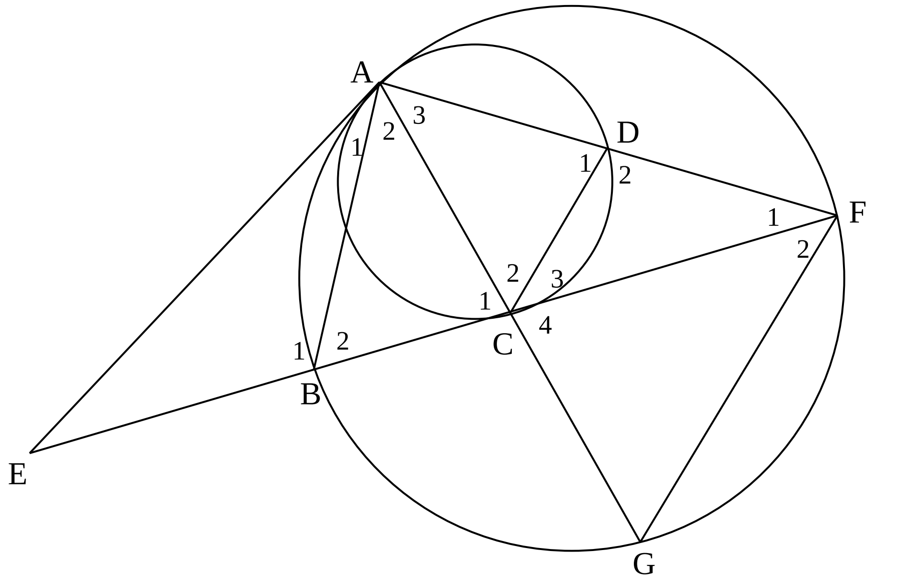
10.2.1	PR = RS PO = OW ∴ OR = $\frac{1}{2}$ WS ∴ OR : WS = 1 : 2	[line from centre ⊥ to chord/ *lyn vanuit midpt. sirkel ⊥ op koord*] [radii / *radiusse*] [midpt theorem/*midpt. stelling*]	✓ S ✓ R ✓ S ✓ S ✓ R (5)
	OR/*OF*		
	$P\hat{S}W = 90°$ $P\hat{R}O = 90°$ ∴ $P\hat{R}O = P\hat{S}W$ ∴ RO ∥ SW	[∠ in semi circle/∠ *in halwe sirkel*] [given] [corresp ∠s = / *ooreenk. ∠e* =] **OR/*OF*** [co-int. ∠s suppl / *ko-binne ∠e suppl*]	✓ S ✓ S
	$\frac{PO}{OW} = \frac{PR}{RS}$ PO = OW ∴ PR = RS ∴ OR : WS = 1 : 2	[prop theorem; RO ∥ SW/ *lyn* ∥ *een sy van* Δ] [radii / *radiusse*] [midpt theorem/ *midpt. stelling*]	✓ S ✓ R ✓ R (5)

	OR/*OF* ΔPRO and ΔPSW $P\hat{S}W = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $P\hat{R}O = 90°$ [given] $\therefore P\hat{R}O = P\hat{S}W$ $\hat{P}$ is common $P\hat{O}R = P\hat{W}S$ [sum of ∠s in Δ/ *som van ∠e in* Δ] $\therefore \Delta PRO \parallel\!\mid \Delta PSW$ [∠∠∠] $\therefore \frac{PO}{PW} = \frac{RO}{SW}$ [||| Δs / ||| Δ*e*] but PW = 2 PO [diameter = 2 radius/*middellyn = 2 radius*] $\therefore \frac{RO}{SW} = \frac{PO}{2PO}$ $= \frac{1}{2}$ $\therefore$ OR : WS = 1 : 2	✓ S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2	$\frac{OV}{VT} = \frac{RS}{ST} = \frac{1}{3}$ [prop theorem; RO || SW/ *lyn || een sy van* Δ] $\frac{RS}{15} = \frac{1}{3}$ $RS = 5$ units PR = RS = 5 units [line from centre ⊥ to chord / *lyn vanuit midpt. sirkel ⊥ op koord*] $\therefore$ PT = 25 units	✓ S /R ✓ S ✓ S ✓ answer (4)
		[15]

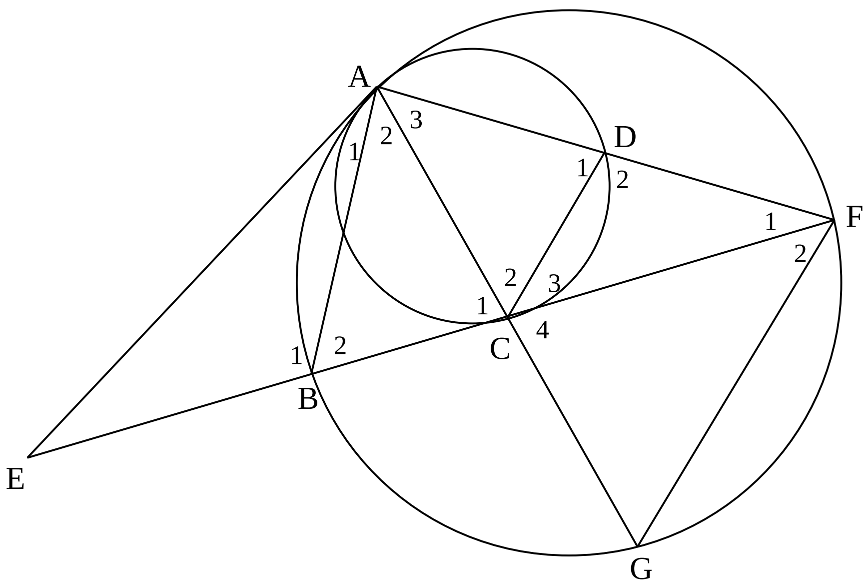
10.2



QUESTION/*VRAAG* 11



11.1	$\hat{D}_1 = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{C}_1 = \hat{D}_1 = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$s = / *regoorst.* $\angle$*e*]	✓ S/R
	$A\hat{F}G = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S (6)
	OR/*OF*		
	EA = EC	[tans from common pt/ *raaklyne vanuit dies. punt*]	✓ S/R
	$\hat{C}_1 = E\hat{A}G = x$	[$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*]	✓ S
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$s = / *regoorst.* $\angle$*e*]	✓ S/R
	$\hat{D}_1 = E\hat{A}G = x$	[tan-chord theorem/ $\angle$ *tussen raaklyn en koord*]	✓ S ✓ R
	$A\hat{F}G = E\hat{A}G = x$	[tan-chord theorem $\angle$ *tussen raaklyn en koord*]	✓ S (6)



11.2	$\hat{D}_1 = A\hat{F}G = x$ $\therefore$ DC || FG [corresp ∠s = / *ooreenk ∠e* =] $\frac{AG}{AC} = \frac{AF}{AD}$ [prop theorem; DC || FG / *lyn || een sy van* Δ] $\therefore$ AG.AD = AC.AF **OR/*OF*** In ΔACD and ΔAGF $\hat{A}_3$ is common $A\hat{F}G = \hat{D}_1 = x$ [proved in 11.1 / *reeds bewys*] $\hat{C}_2 = A\hat{G}F = x$ [sum ∠ Δs/*binne ∠e Δ*] ΔACD ||| ΔAGF [∠∠∠] $\frac{AC}{AG} = \frac{AD}{AF}$ [||| Δs ∴ sides in proportion / ||| Δ*e* ∴ *sye in dieselfde verhouding*] $\therefore$ AG.AD = AC.AF	✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S ✓ S/R ✓ S (4)
11.3	In Δ AGF and ΔABC $\hat{G} = \hat{B}_2$ [∠s in the same seg / *∠e in dies. segment*] $A\hat{F}G = \hat{C}_1 = x$ [proved in 11.1 / *reeds bewys*] $\hat{A}_3 = \hat{A}_2$ [sum of ∠s in Δ/*binne ∠e van* Δ] Δ AGF ||| ΔABC [∠∠∠]	✓ S ✓ R ✓ S ✓ S **OR/*OF*** R (4)

11.4	$\frac{GF}{BC}=\frac{AF}{AC}$ [$\Delta AGF \mid\mid\mid \Delta ABC$] $\therefore GF=\frac{BC.AF}{AC}$ $\Delta ACD \mid\mid\mid \Delta FGC$ [$\angle\angle\angle$] $\therefore \frac{AC}{GF}=\frac{AD}{FC}$ $\therefore AC=\frac{AD.FG}{FC}$ $\therefore GF=BC.AF\div\frac{AD.FG}{FC}$ $GF=BC.AF\times\frac{FC}{AD.FG}$ $\therefore GF^2=\frac{BC.FC.AF}{AD}$ **OR/*OF*** $\Delta AGF \mid\mid\mid \Delta ABC$ [$\angle\angle\angle$] $\frac{GF}{BC}=\frac{AF}{AC}$ $GF=\frac{AF.BC}{AC}$ $\Delta ACD \mid\mid\mid \Delta AGF$ [$\angle\angle\angle$] $\frac{AD}{AF}=\frac{CD}{GF}$ $GF=\frac{AF.CD}{AD}$ $GF\times GF=\frac{AF.BC}{AC}.\frac{AF.CD}{AD}$ $\Delta FCD \mid\mid\mid \Delta FAC$ [$\angle\angle\angle$] $\frac{FC}{FA}=\frac{CD}{AC}$ from $\mid\mid\mid \Delta$\`s $FC=\frac{CD.AF}{AC}$ $GF^2=\frac{AF.FC.BC}{AD}$	✓ S / R ✓ S ✓ S ✓ S ✓ S ✓ S (6) **OR/*OF*** ✓ S ✓ S ✓ S ✓ S ✓ S ✓ S (6)
		[20]

TOTAL/*TOTAAL*: 150